Substitution Practice

Use a trigonometric substitution to integrate the function $f(x) = x\sqrt{x^2 - 9}$. Check your work by integration using the substitution $u = x^2$.

28/8/25

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Use a trigonometric substitution to integrate the function $f(x) = x\sqrt{x^2-9}$. Check your work by integration using the substitution $u = x^2$.

Let $x = \tan\theta$.

$\Rightarrow dx = \sec^2\theta \, d\theta$

$u = \tan^2\theta - 9$

$\Rightarrow du = 2\tan\theta\sec^2\theta \, d\theta$

$$\int x\sqrt{x^2-9}\,dx$$

$$= \int \tan\theta\sqrt{\tan^2\theta-9}\,\sec^2\theta\,d\theta$$

$$= \int \frac{1}{2}\sqrt{u}\,du$$

$$= \frac{1}{2}\left(\frac{u^{\frac{3}{2}}}{3/2}\right) + C$$

$$= \frac{(\tan^2\theta-9)^{\frac{3}{2}}}{3} + C$$

$$= \frac{(\tan^2(\arctan x)-9)^{\frac{3}{2}}}{3} + C$$

$$= \frac{(x^2-9)^{\frac{3}{2}}}{3} + C$$

Model solution substituted for $x = 3\sec\theta$ and then $u = \tan\theta$, and finally inverse substituting back from $\tan\theta$ into terms of x using the Pythagorean theorem.

$u = x^2 \Rightarrow du = 2x\,dx$

$$\int x\sqrt{x^2-9}\,dx$$

$$= \frac{1}{2}\int \sqrt{u-9}\,du$$

$$= \frac{1}{2}\cdot\frac{(u-9)^{\frac{3}{2}}}{3/2} + C$$

$$= \frac{(u-9)^{\frac{3}{2}}}{3} + C$$

$$= \frac{(x^2-9)^{\frac{3}{2}}}{3} + C$$

Model solution checked answer using substitution for $u = x^2-9$ for some reason even though question asked for $u = x^2$.